# Dynamic Flexibility, Optimal Organization Modes and Price Instability

Alfredo del Monte, Francesco Dell'Isola

UNIVERSITÁ DEGLI STUDI DI NAPOLI FEDERICO II

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Alfredo Del Monte *

Francesco Dell'Isola *

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# DYNAMIC FLEXIBILITY, OPTIMAL ORGANIZATION MODES AND PRICE INSTABILITY

M.Fourier avait l'opinion que le but principal des mathématiques était l'utilité publique et l'explication des phénomènes naturels; mais un philosophe comme lui aurait dû savoir que le but unique de la science c'est *l'honneur de l'esprit humain*, et que sous ce titre, une question de nombres vaut autant qu'une question du système du monde

C.G.J.Jacobi

lettre à Legendre

epigraph to Dieudonné 1987


ABSTRACT:

In this paper a mathematical model describing the organization structure of firms (i.e. those economical entities which are called "productive systems" in [14] ) is introduced.

In the framework of quoted model we can give a precise meaning to the classical definition of flexible industrial organizations (Stigler flexibility) which reads as follows:

"a flexible industrial organization permits to approximate the best technology for any output, at the cost of not being able to use the best technology for any output " ( see [2] on pg. 315 ) ,

Moreover we can prove that the presence of exchange price instability enriches the manifold of firms which are competitive. Indeed we characterize the manifolds of equally competitive firms as the level surfaces of the previously introduced profit function. At the same time we introduce the concept of "feasible" organization structures, the set of all of them characterizing the available technology. In this way we are led to a problem of constrained optimization whose solution is in general non-unique: we are thus able to characterize technologies and instabilities which produce a multeplicity of optimal organization structures.

# 1. Introduction

The recent theory of the firm has proposed analytical models that go beyond the black-box conception of production function. Therefore firm's organization has become a very important topic in industrial organization literature. Following transaction cost approach investigations concentrated on the factors that affect the efficiency of alternative forms of firm's organization. Organization forms are affected by economic environment (market size, price stability, technology, etc.) and by internal factors (size of the firms, interdependence with other firms, incentive systems, internal labour market, etc.). Many writers seem to have proceeded that one particular characteristic is the major determinant of the optimal organization. Organizational theorists have stressed environmental uncertainty as the determinant of the optimal organization form (Burns [21]). The organismic system of management is more adapted to unstable conditions when new and unfamiliar problems and requirements continually arise. The mechanistic systems of management, that is characterized by a hierarchic structure of control, authority and communication, is appropriate to stable conditions. Other authors as Piore and Sabel [22] have assumed that the increasing economic environmental uncertainty is the main cause of the success of the flexible-specialization-organization model relative the mass-production model. All these authors assume that in given environment only one organization will prevail, i.e. hierarchy companies will prevail in stable slow-growth industries like oil, paper and forest production, while companies in fast-changing markets like computers, telecommunications, publishing autos, specialty steel will follow a more flexible organization model.

This view is shared by most economic literature that postulate that exist a clear general evolution in each industry towards one organizational form and technology which is considered optimal. Also if optimality argument does not involve necessarily only one solution most economist expect that in a given environment a particular organization structure will predominate. In the standard model of competitive equilibrium production technology and market demand determines the size and organization structure of the firms, all identical. According to Chandler [23] and to Williamson [24] among the possible structures of internal organization, the ones tending to predominate over time are those that insure the minimization of production and transaction cost. Also authors (Aoki [25], Milgron-Roberts [26]) that have built analytical models that incorporate different features of organization forms had as main interest to compare the efficiency of different models.

The purpose of our paper is to analyse the effect of output price instability on the technological and organizational choice of a risk neutral competitive firm which faces adjustment costs to change from one rate of output to a different one.

Two are the dynamic element in our model. The first is a time dependent price level. The second is the introduction of dynamic adjustment cost: more precisely we assume that, at time t, an organization mode F could be described by a dynamic cost function that has the following form:

$$F := C[q(t),\dot{q}(t)]$$

This means that at a given time instant t, the cost of producing the quantity q will depend on the instantaneous value of q and on the speed of variation of q: this last dependence will take into account the adjustment cost.

The aim of this paper is to propose a mathematical model to describe the choice of the optimal organizational mode in a market in which price oscillations occur.

The choice of optimal mode has to be done in two steps.

The first step is to find the optimal path $q^*(t)$ for a given organization mode F. This is the path which maximizes the profits of F in the interval [0,T]. We call $q^*(t)$ the *optimal tactics* of the organization mode. The second step is to choose among the feasible organization modes those which allow the greatest profit. We call industrial strategy the choice of a given F.

The result of the paper is that *if the isoprofit curves and the set of feasible organization modes are both convex and if the convexity of the second is higher than that one of the first* then it is possible to prove that there exists one and only one *optimal organization mode*,i.e. the mode which is the most profitable. We also prove that this optimal mode is changing as price oscillation function changes. Moreover we are able to prove that as time instability of price increases (this time instability being measured by a suitable norm in the space of price oscillations) the number (i.e. the measure of its representative set in the space of organization modes) of organization modes that realize positive profits increases.

We stress that quoted uniqueness theorem *does not hold* if aforementioned relative convexity hypothesis is not verified. In the conclusions we underline that it is conceivable, from an economical point of view, a technology and a profit function which do not verify it: in these cases more than one equiprofitable optimal organization modes *will coexist.* Finally we remark that a by-product of this result is the extension of the concept of *Stigler flexibility* to a dynamic context.

# 2. The Mathematical Model: the case of a competitive industry

In this section we introduce the mathematical concepts we need in order to model the behavior of a large class of a single-product firms in presence of periodical variations of price p of the produced good.

Moreover we will assume that firms exchange the good in a competitive industry.

We assume that

i) the price p cannot be influenced at all by any of the firms producing the same product. However, due to an intrinsic market instability, in different instants t belonging to a given time interval [0,T] the product is exchanged at different prices. This mathematically means that it is possible to define a price function of the variable t:

$$p : t \in [0,T] \longmapsto p(t)$$

We will assume that the time interval [0,T] represents a periodicity interval for price oscillations, so that the following equality holds:

$$p(t) = \langle p \rangle + \Delta p(t)$$

where $\Delta p(t)$ is a suitably regular function which vanishes in both instants 0 and T.

ii) the production technology of F can be characterized when F chooses a stationary production regime by means of its Cost function ( see e.g. [7] ).

However, in order to take into account the economic phenomena described in [1], we have to implement the classical approach of the micro economic theory of the firm considering, together with the "stationary" production Cost function, another Cost function, which we call *adaptation Cost function.*

To this aim let us introduce the following definitions:

## 2.1 PLAN OF PRODUCTION

We call plan of production of the firm F in the period [0,T] a real valued function q:

$$q : [0,T] \longrightarrow \mathbb{R}^+$$

such that

$$q : t \in [0.T] \longmapsto q(t)$$

where q(t) is the quantity of the good produced in unit time at the instant t.

## 2.2 Dynamic Cost Functions

We assume that an organization mode F could be described by a dynamic cost function that has the following form:

$$F := C[q(t),q(t)]$$

This function allows us to represent together with the static cost also the extra cost met by F in every time interval [0,T] because of the time variation of the produced quantity q. This cost, we assume to be determined once the speed of change $\dot{q}$ is known, is called "adjustment" or "adaptation" cost.[1]

When a firm has to change its rate of output it will bear a "once for all" costs. These costs do not necessarily involve exclusively expenses due to the need of adding new capital but mainly expenses of "software type". The main factors that cause adaptation costs are the following:

a) the loss of production in the time needed to reschedule the plant or the machines to the new rate of activity

b) the cost of the new rescheduling plan and the cost of training workers to the new plan (Treadway [31], Uzawa [32])

c) the cost of bargaining, in a world where labour is a quasi fixed factor, with trade-unions the new organization of labour (change the number of shifts per day, change the number of working hours per worker, etc.)

d) the negotiation costs with subcontractors relative the different rate of outputs that they must supply.

The adaptation costs could be asymmetric (the same rate of increase or decrease of output production could produce different variations of costs) but in the present paper we will consider only the symmetric case.

We will also assume that average adaptation costs increase as the rate of change of output, in absolute terms, increases. In our view increasing average adaptation costs are meanly caused by factors c and d. When the rate of change of output is not very high and the effect on organization of labour is not very low, bargaining costs will be very low, but they will increase if the rate of change of output is quite high. The same will apply with negotiation costs with subcontractors.

Our hypothesis is that an organization mode is comprehensive of a static cost function and an adaptation cost function, so that to its stationary cost we have to add its adaptation cost.

We will limit our attention to the case in which firms in an industry can choose an organization mode from a set of continuous options each of the following type:

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(1) Most of the literature (for instance R.Hartman [27], R.Pindyck [28], A.Abel [29], P.Caballero [30]) focused on the relationship between uncertainty and the rate of investment. In quoted papers adjustment costs are introduced, however referring to the cost of changing the stock of capital. In our paper, instead, we refer to the cost of changing the rate of output and we call these costs "adaptation costs".

$$C:\ q \longmapsto C(q) = c_0\, q + \frac{c_1}{2}\, q^2 + c_2 + \frac{c_3}{2}\, \dot{q}^2 \qquad (2.1)$$

The economic meaning of the parameters is the following:

- the parameter $c_0$ represents the price threshold for F, i.e. the price of exit, in stationary regimes, of an incumbent firm;
- the parameter $c_2$ represents so called fixed costs for F
- the parameter $c_1$ represents the second derivative of total stationary costs with respect the produced quantity; the reciprocal of $c_1$ is a measure of static flexibility (cf. Marschak T.-Nelson R. [33])
- the parameter $c_3$ represent the adjustment costs relative to a unitary speed of variation of output production.

We will call *linear* (2) *differential single-product firm* a firm whose cost function belongs to the set of functions of type (2.1).

Therefore the single firm belonging to the considered set is obviously characterized by means of the set of positive real parameters

$$\pi := \left\{ c_i\ ;\ \ i = 0,1,2,3 \right\}. \qquad (2.2)$$

We say that two firms are of the same *kind* if they are characterized by the same set $\pi$.

It is clear that, in whatsoever technological situation, not all cost functions are feasible: the zero cost function (that one in which all c's are vanishing) and those which are suitably close to it will never be possible. Technological progress can make possible some organizational modes which before were impossible: however in every state of the art some cost functions are unattainable. Therefore the adaptability possibilities, when an arbitrarily fixed technology is considered, have to be regarded as constrained by the limits of the technology itself.

We will model this circumstance introducing the *technological constraints*, represented by inequalities which the coefficients appearing in the cost functions must verify in a given technological situation

$$T_k(\pi) \geq 0 \qquad k=1,..,h \qquad (2.3)$$

where h represents the number of inequalities necessary to characterize the set $\Phi$ of feasible organizations.

(2) The reasons for this adjective will become evident in the next section.

We will say that the functions $T_k$ characterize the available technology.

We will assume that $\Phi$ does not include a neighborhood of the point $\mathbf{0} := (0,0,0,0)$, and that it is the closure of an open set. In figures 1 to 4 the intersections of some sets $\Phi$ with the planes $c_i c_j$ are the parts of the first quadrant on the upper side of the bold line 1.

## 2.3 Profits and Prices

2.6.1) The total profits G(T) made by the linear differential single-product firm F in the time interval [0,T], are obtained by means of the following formula, which expresses the *short range* profit functional

$$G(T) = \int_0^T e^{-r\tau} \left[ p(\tau)q(\tau) - c_0 q(\tau) - \frac{c_1}{2} q^2(\tau) - c_2 - \frac{c_3}{2} \dot{q}^2(\tau) \right] d\tau \tag{2.4}$$

where :

$q(\tau)$, $p(\tau)$ respectively represent the plan of production chosen by F and the exchange price oscillation in the time interval [0,T], and $r$ denotes the discount rate in the period [0,T].

We remark that G depends also on $\pi$ and the function p(t).

2.6.2) In this paper we assume that the exchange price function p(t) is completely known in the time interval [0,T], so that the plan of production q(t) can be adjusted in the whole time interval [0,T] in a completely determined optimal way. A lack of knowledge implying the possibility of forecasting only some features of the function $p(\tau)$ in the subsequent instants could be accounted, in our opinion, using more sophisticated approaches, following for instance the treatment found in [5].

2.6.3) The set $P_p$ of profitable organizations in presence of the price oscillation function p(t) is characterized as follows:

$$P_p := \left\{ \pi \in \Phi \ / \ G_{\pi,p} \geq 0 \right\} \tag{2.5}$$

In Fig.1 are drawn two sets P, bounded by the curve 1 and respectively by the curves 2 or 3. They represent the set of profitable organizations in the presence respectively of a constant exchange price $p^*$ or of time varying price whose average is $p^*$.

## 3. Optimal Plan of production For Linear Differential Firms.

The problem that we want to solve could be stated in the following way:
for a given organization mode $\pi$, a given price oscillation p(t) and a plan of production q(t) a given level of profit G is fixed by (2.4), so that we could state that

$$G = G(\pi,p(t),q(t)).$$

The first step of our analysis is to find for a given $\pi$ and p(t) the optimal plan of production $q^*(t)$ i.e. the plan for which G is maximum. This optimal value we will call $G^{op}$.
The second step is to find, inside the set $\Phi$, those organization modes which correspond to the maximal $G^{op}$. This maximum we will call $G^*$. We will find the conditions in which only one organization mode exists which corresponds to $G^*$.

Let us start from step one: assuming completely known in the time interval [0,T] the exchange price function p(t), which is the optimal plan of production to be chosen by F ?
Calculus of variations supplies us the techniques we need to settle down this question.
Indeed (see for instance [3],[11] and [20]) the functionals (2.4) defined in the set of feasible plan of production for F assume their optimal values when evaluated in the solutions of Euler-Lagrange Equation related to their integrand Lagrangian function. Simple calculations lead us to the following

### Proposition 3.0

Let us consider the set $\Sigma$ of all plan of production in $C^2[0.T]$ which assume the assigned boundary values

$$q(0) = q_0 \ ; \ q(T) = q_f \tag{3.0}$$

The optimal plan of production for a given organization mode in $\Sigma$ with respect to the functional (2.5) is the solution of the following ordinary linear second order differential equation:

$$\mathcal{A}q := \alpha_1 \frac{d^2q}{dt^2} - \alpha_2 \frac{dq}{dt} - \alpha_3\, q \ = \ -\,(p(t)-c_0) =: k_0(t) \tag{3.1}$$

where the positive constants $\alpha_i$ are expressed in terms of the constants r, $c_i$ by means of the following equalities:

$$\alpha_1 = c_3 \ ; \ \alpha_2 = rc_3 \ ; \ \alpha_3 = c_1$$

Proposition 3.0 accounts for the Definition 2.4: indeed the differential operator $\mathscr{L}$ defined in Eq.(3.1) is linear.

We consider now the set $\Pi(T)$ of all periodic continuously differentiable functions whose period is T and its subsets

$$\Pi_0^i(T) \ ( i \in \mathbb{N} - \{0\} )$$

of all functions i times continuously differentiable whose boundary values ( in the [0,T] interval) are vanishing.

We are interested to these subsets because we will use a function belonging to them in order to model price instabilities.

## Definition 3.1 Market Instability

We will call *market instability* a function belonging to $\Pi_0^2(T)$.

It will be the sum of such a function with a constant one which will appear at RHS in Eq.(3.1).

As it is well known ( see for instance [12] ) it is possible to represent every $f \in \Pi(T)$ by means of trigonometric series: i.e. considering their Fourier series with respect to the $L^2$ basis

$$\left\{ 1 \ , \ \cos\left(i\omega t\right) \ , \ \sin\left(i\omega t\right) \ ; \ i \in \mathbb{N} - \{0\}, \ T\omega = 2\pi \right\}.$$

However this basis is not the most suitable if one wants to evaluate the images assigned by the functional (2.5) to the optimal plan of production.

Indeed we will prove that the most useful representation for the functions belonging to $\Pi_0^i(T)$ is obtained when considering the Fourier series with respect to a special set : that which collects the eigenfunctions solution of Sturm-Liouville problem for the linear operator $\mathscr{L}$ defined in $\Pi_0^2(T)$.

## Proposition 3.1

The linear operator

$$\mathscr{L} : \Pi_0^2 \longrightarrow C^0([0,T])$$

is a Sturm-Liouville operator. The set $\mathscr{EF}$ of its eigenfunctions

$$\left\{ 1, \ \exp\left(rt/2\right)\sin\left(i\omega t/2\right); \ i \in \mathbb{N} - \{0\} \right\}$$

is such that:

i) Let $f \in \Pi_0^1$. Its Fourier series with respect to $\mathscr{EF}$ is uniformly absolute convergent to f itself;

ii) the eigenvalue

$$a_i := c_1 + c_3\left(i\omega/2\right)^2 + c_3 r^2/2 \tag{3.2}$$

relative to the eigenfunction

$$\exp\left\{rt/2\right\}\sin\left(i\omega t/2\right)$$

has multiplicity one;

iii) the set of eigenfunction $\mathcal{EF}$ is orthogonal with respect to the inner product

$$\left(u,v\right) = \int_0^T \exp\left\{-rt\right\} u(t)v(t)\, dt$$

Proof:

All i), ii) and iii) can be obtained by means of direct computations using the results found in [12] and [16]. However a better insight in the mathematical structure of the model we are introducing is obtained proving these statements by means of the results found in [15].

Indeed when recalling formula (7) on pg.40 in [15], by means of the following identification ( the variable x has to be identified with the variable t )

$$p(x) = \exp\left\{-rt\right\}\ ;\ k(x) = \exp\left\{-rt\right\};\ q(x) = \alpha_3 \exp\left\{-rt\right\}$$

we can prove that our operator $\mathcal{L}$ is a Sturm-Liouville operator.

Moreover the ordinary differential equation

$$\mathcal{L}u = \lambda u$$

has the following general solution:

$$W(t) = C\exp\left\{rt/2\right\}\sin\left(wt\right) + D\exp\left\{rt/2\right\}\cos\left(wt\right)$$

being w one-to-one correspondent to $\lambda$. In order to be assured that W(t) belongs to $\Pi_0^2$ the following relations must hold:

$$D = 0\ ;\ w = k\pi/T \quad \text{where } k \in \mathbb{N}.$$

The first between these relations proves that the multiplicity of all eigenvalues of $\mathcal{L}$ is one, while simple calculations lead to the expression (3.2).

Finally the proof is easily obtained recalling the Theorem on pg.125 in [15].

Proposition 3.1 allows us to represent the function $k_0 \in \Pi(T)$ appearing at RHS of Eq.(3.1) as follows:

$$p(t) = \langle p\rangle + \sum_{i=1}^{\infty} P^i \exp\left\{rt/2\right\}\sin\left(i\omega t/2\right) \tag{3.3}$$

where $\langle p\rangle$ denotes the average of p(t) in [0,T].

Therefore the perturbation with respect to the average can be represented by means of what we will call the *perturbation* vector belonging to $\ell^2$ ( cf. for instance [12] )

$$A := \left(P^i\right)$$

We can now use the Fourier Method for solving Eq.(3.1):

## Proposition 3.2

The unique solution of equation (3.1), when $k_0(t)$ is given by equation (3.3), which verifies the condition [(3)]

$$\lim_{\|A\| \longrightarrow 0} q(A,t) = \frac{<p> - c_0}{c_1} \tag{3.4}$$

is given by the following expression

$$q(t) = \frac{<p> - c_0}{c_1} + \sum_{i=1}^{\infty} \exp\{rt/2\} \left(Q^i \sin\left(i\omega t/2\right)\right) \tag{3.5}$$

where

$$Q^i = \frac{P^i}{a_i} \tag{3.6}$$

Proof:

We use the methods developed in [17] proving, in the considered instance, the Principle of the Superposition of Effects. For a detailed discussion about the range of validity of this Principle and its use in applied mathematics we refer to [18].

First we remark that, because of linearity of Eq.(3.1), simple algebra leads to Eq.(3.5) when the vector **A** has only a finite number of elements which are not vanishing.

For this reason we consider the series formally represented by Eq.(3.5): as the vector **A** belongs to $\ell^2$ and the sequences

$$\left\{a_i\right\}_{i\in\mathbb{N}} \quad \text{and} \quad \left\{2a_i\left(i\omega\right)^{-1}\right\}_{i\in\mathbb{N}}$$

tend to $+\infty$ , then the vectors

(3) Equation (3.4) represents a re-formulated conditions (3.0) when both $q_0$ and $q_f$ are assumed to be equal to $\dfrac{<p> - c_0}{c_1}$

$$\mathbf{B} := \left(Q^i\right) \; ; \; \mathbf{C} := \left(i\omega Q^i\right)$$

also belong to $\ell^2$.

This means that RHS of Eq.(3.5) is meaningful and that the series resulting from its formal derivative (term by term) also is convergent. Moreover as the sequence

$$\left\{\left(i\omega\right)^2 a_i^{-1}\right\}$$

is bounded, the vector

$$\mathbf{D} := \left(\left(i\omega\right)^2 Q^i\right)$$

belongs to $\ell^2$, so that also the second order formal derivative of the series (3.5) is convergent.

The proof is finally completed recalling that

i) the classical theorem about the convergence of formal derivatives assures the existence of first and second order derivatives of the RHS of (3.5);

ii) every partial sum of the series on the RHS of Eq. (3.5) is a solution of Eq. (3.1) when p(t) is substituted by the corresponding partial sum from the series on the RHS of Eq.(3.3).

We remark that:

i) $Q^i \leq \dfrac{P^i}{c_1}$ ; where the quality holds if and only if $c_3=0$

ii) Equation (3.4) mathematically formulates the following economic requirement : when the perturbation with-respect-to-the-average term in (3,3) vanishes the optimal plan of production for F is the constant one, and the constant produced quantity has to be determined ( see for instance [7] ) either equating marginal cost MC to exchange price p or equating marginal profits MP to zero..

iii) When $c_3 = 0$ the equality MC = p ( or MP = 0 ) holds at every instant : the firm has no adaptation extra-cost (and a supply and demand law unaffected by "expectations" holds) so that it can instantaneously maximize profits. In this case our model becomes exactly that proposed in [1].

We can therefore introduce the quantity $s_i$ which measures the amplitude (of oscillating production function) ratio between the "ideal" case in which $c_3$ is vanishing and the actual case where this constant cannot be neglected:

$$s_i c_1 := a_i \qquad (3.7)$$

We are now ready to introduce our optimal profits function G: it is a function defined in the

Cartesian product

$$\Phi \times \mathcal{Z} \times \mathbb{R}$$

whose range is $\mathbb{R}$. G maps the vector

$$\left(c_i, \ \mathbf{A}\ , \ \omega\right)$$

to the optimal profit $G^{op}$ which can be obtained by F during the time interval [0,T].

## Proposition 3.3

Under all hypotheses assumed up to now, the following representation formula for the optimal profit function $G^{op}$

$$G^{op}/T = \frac{\left(<p> - c_0\right)^2}{2c_1} - c_2 + \sum_{i=1}^{\infty}\left(\frac{\left(p^i\right)^2}{4a_i}\right) \tag{3.8}$$

holds.

We remark that the *stationary part of profits* is given by the formula

$$G_s/T := \frac{\left(<p> - c_0\right)^2}{2c_1} - c_2 \tag{3.9}$$

Proof of Proposition 3.3:

In order to obtain (3.8) it is enough to substitute the optimal plan of production (3.5) in formula (2.5) and perform the integration. This last operation is carried on using simple algebra and recalling:

i) the $L^2$-orthogonality of the set of functions

$$\left\{ \sin\left(i\omega t/2\right),\ i \in \mathbb{N} - \{0\}\right\},\ \left\{\cos\left(i\omega t/2\right), i \in \mathbb{N} - \{0\}\right\}$$

and the expression of their $L^2$ norms.

ii) the Lebesque's bounded convergence theorem ( cf. for instance [19]).

We remark that a simple reasoning, similar to those developed in the proof of Proposition (3.2), can lead to directly prove that the series on the RHS of Eq.(3.8) is convergent.

## 4. Stigler Flexibility : Competitive and Optimal Firms.

We remark that, for a given organization mode $\pi$, the choice of the optimal (time dependent) plan of production instead of a stationary one implies some additional profits which in terms of the price oscillation are given by the following formula

$$\mathcal{SF} := \sum_{i=1}^{\infty} \left( \frac{\left(P^i\right)^2}{4a_i} \right) \tag{4.1}$$

Thus we are led to call, generalizing the definition introduced in [1], *Stigler Flexibility* (with respect to the i-th frequency of price oscillation) of a Linear Differential Firm the quantity

$$\frac{1}{4a_i}$$

This quantity measures the capability of F of making extra-profits choosing instead of a constant plan of production the optimal time variable plan of production determined in the previous section.

*Stigler flexibility is always positive:*
this means that in markets with price instability it is possible to find competitive firms which were not competitive in the absence of price oscillations.

We explicitly remark that:

i) When $c_3$ is vanishing then formula (4.1) becomes:

$$\mathcal{SF} := \frac{1}{4c_1} \sum_{i=1}^{\infty} \left(P^i\right)^2 \tag{4.2}$$

which, when we recall that the quantity

$$\sum_{i=1}^{\infty} \left(P^i\right)^2$$

represents the variance with respect to the average of price oscillation (3.3), coincides with that obtained in [1].

ii) Stigler Flexibility attains its maximum value when $c_3$ vanishes. Moreover it is a decreasing function of the parameter $c_3$ and when $c_3 \longrightarrow \infty$ then it is vanishing.

iii) Ceteris paribus, Stigler Flexibility is a decreasing function of the pulsation $\omega$. This result, while obvious from an economic point of view, cannot be obtained using the approach developed in [1]: indeed in the model which is proposed there no mathematical concept is available to describe in a detailed way the *history* of price oscillation.

iv) The following problem could arise: when a periodic price oscillation more general than (3.3) is considered is it possible to apply a generalized form of Fourier method in order to determine optimal plan of production and optimal profits for linear differential firms ?

It seems to us that such a generalization is possible only considering weaker topologies in $\Pi(T)$ than that one determining uniform convergence. To this aim the results available in [12] could be useful.

Let us now fix the price oscillation p(t):

we say that two firms, respectively determined by means of the parameters $\pi^1$ and $\pi^2$ are *equiprofitable* in presence of the oscillation p(t) if and only if

$$G(\pi^1) = G(\pi^2) \tag{4.3}$$

Therefore the relation

$$G(\pi) = G_0 \tag{4.4}$$

determines the set $\mathcal{E}$ of all equiprofitable (in presence of the market instability $k_0$) firms whose profits are $G_0$.

Due to Dini's Theorem equation (4.3) determines an embedded submanifold $\mathcal{V}$ of $\mathbb{R}^4$, i.e. an hypersurface of points belonging to $\mathbb{R}^4$. The dimension of such a surface is three.

Once fixed $c_0$ and $c_2$, i.e. the parameters which do not influence the Stigler Flexibility of considered firm, all equiprofitable firms lay on the level curves ( in the plane $(c_1,c_3)$ ) of the so obtained partial optimal profits function.

These curves can be regarded as the intersection of $\mathcal{V}$ with the plane $(c_1,c_3)$.

In this way we have proved the

## Proposition 4.1

*If there is market instability it exists a three-dimensional manifold of equally profitable organization modes. Moreover for any couple of coefficienys of price threshold and fixed cost still there exists a one dimensional manifold of equiprofitable firms.*

It is possible to find equiprofitable firms among those whose stationary profit is lower and whose Stigler Flexibility is higher or vice versa. Moreover also among those firms whose fixed cost and price thresholds are equal, one can find equiprofitable firms whose Stigler Flexibility and stationary profits are different.

We are now ready to prove, in the framework of proposed model, the conjecture made in [1] and investigated more deeply in [13].

In a mathematical language it can be formulated as follows:

i) *The measure of the set of profitable organizations increases in presence of price* *instabilities.*

Indeed if to a constant price $<p>$ we add a price oscillation whose average is vanishing, because of the positivity of Stigler flexibility the hypersurface of equiprofitable firms which obtain exactly zero profits moves upwards (e.g. from curve 2 in fig.1 moves to curve 3).

Let us now draw our attention to Fig.2.

The curve 1 there represents the intersection of the boundary of the set $\Phi$ with the plane $c_i c_j$. We will use the denotation

$$\gamma := \partial\Phi \cap \{c_i c_j\}$$

ii) *If the convexity of* $\gamma$ *is higher than that of the isoprofit level curves then there exists an unique optimal organization for every price oscillation.*

The analytical proof of ii) would require the use of some notions of differential geometry: because of the immediate geometrical meaning of the concepts involved in it, and due to the possibility of analysing the problem using bidimensional pictures we refer to Fig.2. In it $O_1$ represents the optimal feasible organization, belonging to the set of equiprofitable organizations represented by the curve 3. We remark that other organizations are profitable. If Curve 2 represents the zero profits equiprofitable set of organizations, the regions bounded by curve 1 and curve 2, i.e. the curvilinear triangle $ABO_1$ represents the set P in the considered instance.

iii) *a sufficient condition to have in competitive equilibrium a unique optimal organization is that the convexity of* $\gamma$ *is higher than that of isoprofit curve* $G(\pi)=0$.

However the curve $\gamma$, always curve 1 in both Fig.3 and 4, can be either less convex than isoprofit curves or even *not convex*.

We can prove now that,

iv) *convexity of* $\gamma$ *is not a sufficient condition for the existence of a unique optimal organization*

Indeed (see Fig.3) increasing the profit k the isoprofit curve moves downwards, passing continuously from the situation represented by the curve 2 (in which feasible organizations are lower than D and higher than C) to the situation represented by the curve 3. The organizations A and B are both equiprofitable and optimal.

A similar reasoning holds if $\gamma$ is not convex, as for instance shown in Fig.4, where the two optimal and equiprofitable organizations are $O_1$ and $O_2$.

# 5. The Convexity Properties of the Set of Feasible Organization Modes

When we allow instability in the price level economic experience do not allow us to say much on the set of the coefficients of the cost curves.

Indeed in static regimes, for each q we observe only the organization modes whose parameters $\pi_s := (c_0, c_1, c_2)$ minimize total stationary costs. Under the standard assumptions about technology (Varian [7]) and fast decreasing and then increasing return to scale, we will have for every q only one cost function which minimizes total cost. When we allow instability, the same reasoning could not be done as flexible plant could be preferred to less flexible ones, also if for each value of q is possible to find plants that show lower total minimum costs (Del Monte-Esposito [1]). Therefore the set of feasible cost functions that are economically relevant is much larger than in the static case and probably not convex. However also in a static world empirical evidence could help us to establish that the set of feasible $\pi$ may not be convex: the examples of organization modes which we will describe in what follows and which can be found in different industrial sectors such as textiles, semiconductors, personal computers and also in steel would help to clarify our point.

Indeed, in these sectors, at least three strategies could be chosen by a firm in order to establish its organization mode.

First it could built a plant specifically dedicated to the production of a desired quantity of output. An example is the typical Fordist plant that is highly mechanized and endowed with a relatively high level of working capital. It is able to minimize, in comparison to other strategies, the cost of producing the output at the planned level and composition.

The second strategy would be to build a plant endowed with flexible automation, which, with the help of micro electronics, would allow to, in a very short time and with a relative small costs, change output level and composition. The acquisition cost of such plants is higher than that of the first strategy, as they are more capital intensive.

In the third case, flexibility is achieved through choosing a network organization based on tight and frequent relations with suppliers and buyers. If q represents the processed quantities of outputs and $C_i(q)$ represents the monetary value of both physical and organizational inputs that are required to the i-th element of the network to produce its outputs, then the total cost function

$$C := \sum C_i$$

would have to be characterized by lower capital requirements. Greater flexibility is achieved by increased reliance on variable factors of production, as in the model of Mills and Schumann, so that it is possible to apply "Stiglerian Flexibility" to the network regarded as a unique firm.

The second strategy is characterized by a level of fixed costs higher than the two other strategies. Flexibility is reached through large investment. Here a positive relationship will exist between fixed cost and flexibility: i.e. higher the fixed cost is lower is the adaptability cost. The first strategy is characterized by the lowest average total cost of the fixed quantity of output and by fixed costs which are lower than those implied in the second strategy.

The third strategy is characterized by the lowest fixed costs, by high flexibility and a total average cost higher than that implied by the first strategy.

Now the problem arises to translate in our mathematical language all considerations developed up to now in this section, in order to take them into account when using our model. This is not so easy, also if we restrict ourselves to linear differential firms.

However it is possible to argue that the set of feasible $\pi$ can be non convex in some particular cases of industries: those in which at the same time coexist all three organization modes we have considered up to now. Indeed, if we indicate with 1,2,3 respectively the fordist, flexible automation and the network organization, we are now able (also taking into account the previous considerations) to establish the following

*Relationships between fixed cost and adjustment cost*

$$c_2^2 > c_2^1 > c_2^3 \tag{5.1}$$

$$c_3^1 > c_3^2 > c_3^3 \tag{5.2}$$

We first examine the relationships between cost functions characteristic of strategy 1 and strategy 2. In a static world a feasible organization mode i is an efficient one if at least for one value $q^*$ of output level, the corresponding production cost $C_i(q^*)$ is the lowest possible among all other feasible modes. In formulas

$$C_i(q^*) \leq C_j(q^*) \quad \forall\, j \neq i \tag{5.3}$$

This implies that, together with the first of relationships (5.1), the following inequalities hold

$$c_1^2 < c_1^1 \tag{5.4}$$

$$c_0^1 < c_0^2 \tag{5.5}$$

Indeed the relationships $(5.1)_1$,(5.4) and (5.5) allow that at least for one couple

$$(q_1,q_2) \in \mathbb{R}^+ \times \mathbb{R}^+ \ / \ q_1 < q_2 \tag{5.6}$$

the following relationships (which imply that for different output levels the most efficient mode is different) hold:

$$C_1(q_1) < C_2(q_1) \; ; \; C_2(q_2) < C_1(q_2) \tag{5.7}$$

Then we examine the relationships between strategy 1 and strategy 3. Following the same kind of reasoning the relationships among the coefficients belonging to the two organization modes that satisfy the above mentioned efficiency criterion are

$$c_2^3 < c_2^1 \; ; \; c_1^3 < c_1^1 ; \; c_0^1 < c_0^3 \tag{5.8}$$

In this way we see that all (5.1) hold.

If further we make the assumption, for which there is empirical evidence, that static flexibility is larger in the case of flexible automation than in the case of network firm then the following relation

$$c_1^2 < c_1^3 < c_1^1 \tag{5.9}$$

assures that each of the organization modes will be the most efficient in a certain interval of q.

All relationships we have obtained together imply that the set of feasible organization modes, in the considered industry, is non convex. In fact if we assume that the boundary of the set of feasible modes is represented in the space of $\pi$ by a regular connected surface $\Theta$, the continuous curve which is the intersection of $\Theta$ with the plane of $(c_1,c_2)$ will represent the relationship between efficient static flexibility and fixed cost coefficients. Because of (5.1) and (5.9) this boundary curve, and therefore the set of feasible $\pi$, is non convex.

We can conclude that also in a static world empirical evidence shows that the probability that feasible organization modes is non convex is quite high.

If we further introduce the forth coefficients $c_3$ we will increase the possibility of non convexity, once relationships (5.2) are taken into account.

Therefore we will expect that, at least when the industries described in this section are considered, feasible modes form a non convex set, and consequently it will coexist more than one optimal organization mode.

# 6. Optimal Organization Mode under Monopoly

The model we have built up in this paper need to be improved in order to analyse the choice of the optimal organization mode by a monopoly.

Indeed it is clear that our model, at this stage, does not accounts for the possibility for F to have a stocking policy or to choose a long term strategy whose aim could be the bankrupt of the competitors: in other words in the framework of the present model no effort is done to describe in detail all those economic phenomena which occur in a Market while a Monopoly establishes its control upon it.

On the other hand we can try to find the set of all technologies which yield the optimal profit assuming that the Monopoly already has reached its dominant position.

## 6.1 Instability in Demand Law

If F is a monopoly the price $p(\tau)$ is determined by means of the following demand law with expectation:

$$p = k_0 - k_1 q - k_2 \dot{q} \text{ [4]} \tag{6.1}$$

where $k_i$ ( $i = 0,1,2$ ) are given real positive functions of time:

when $i = 1,2$ $k_i$ models the reactivity of market to variations of produced ( and therefore exchanged, as we are in a monopolistic Market ) quantity q, while $k_0$ is introduced in order to account price oscillations which cannot be controlled by F.

We explicitly remark here that the case in which F exchanges its product in a competitive industry formula (6.1) is still valid: indeed when the functions $k_1$ and $k_2$ are both vanishing, it results that the firm F cannot influence at all the exchange price, whose oscillations, however, still occur and are accounted by the function $k_0$.

*We will call the triple of functions* $k_i$ *Market Instability.*

However, in what follows, only the function $k_0$ will be assumed to be time-dependent (both $k_1$ and $k_2$ being constant with respect to time): this hypothesis will allow us to simply generalize the results of previous two sections to the considered case. The treatment of the more general case will be attempted in [9].

The problems (similar to those confronted in sect.3) now arise:

i) assuming completely known in the time interval [0,T] the Market Instability function $k_0(t)$, which is the optimal plan of production to be chosen by F ?

---

(4) This relation parallels formula (15.31) on pg.489 in [10]. The upper dot denotes derivation with respect to time variable.

ii) how we have to define the Stigler Flexibility of F ?

We begin remarking that Proposition 3.0 still holds, if $k_0$ is assumed to play the role of the function p(t) and the coefficients $\alpha_i$ are defined as follows:

$$\alpha_1 = c_3 \ ; \ \alpha_2 = rc_3 \ ; \ \alpha_3 = rk_2 + k_1 + c_1$$

Moreover we will keep in assuming that *market instabilities* are functions belonging to $\Pi_0^2(T)$ added to some constants.

Also Proposition 3.1 can be equally be proved also in the present instance, when the eigenvalue $a_i$ is expressed as follows:

$$a_i := c_1 + c_3\left(i\omega/2\right)^2 + c_3 r^2/2 + k_1 + rk_2 \tag{6.2}$$

Therefore for the function $k_0 \in \Pi(T)$ the following representation is possible:

$$k_0(t) = \langle k_0 \rangle + \sum_{i=1}^{\infty} P^i \exp\left\{rt/2\right\} \sin\left(i\omega t/2\right) \tag{6.3}$$

where $\langle k_0 \rangle$ denotes the average of $k_0$ in [0,T], so that the *perturbation* vector

$$A := \left(P^i\right)$$

can also be introduced.

In this way we can generalize also Proposition (3.2) which can now be formulated as follows:

## Proposition 3.2'

The unique solution of equation (3.1), when on RHS appears the function $k_0(t)$ (represented by means of equation (6.3)), which verifies the condition

$$\lim_{\|A\| \longrightarrow 0} q(A,t) = \frac{\langle k_0 \rangle - c_0}{k_1 + c_1} \tag{6.4}$$

is given by the following expression

$$q(t) = \frac{\langle k_0 \rangle - c_0}{k_1 + c_1} + \sum_{i=1}^{\infty} \exp\{rt/2\} \left(Q^i \sin\left(i\omega t/2\right)\right) \tag{6.5}$$

where

$$Q^i = \frac{P^i}{a_i} \tag{6.6}$$

We remark that:

i) $Q^i \leq \dfrac{P^i}{c_1 + k_1}$ ; where the quality holds if and only if $c_3 = k_2 = 0$

ii) the ratio $s_i$ is now defined as follows:

$$s_i(k_1 + c_1) := a_i \tag{6.7}$$

iii) the optimal profits function G is now a function defined in the Cartesian product

$$\left(\mathbb{R}^+\right)^6 \times \mathbb{Z} \times \mathbb{R}$$

with range $\mathbb{R}$.

The following representation formula for the optimal profit function G

$$G/T = \frac{\left(<k_0> - c_0\right)^2}{2(c_1 + k_1)} - c_2 + \sum_{i=1}^{\infty} \left( \frac{\left(P^i\right)^2}{4a_i} \right) \tag{6.8}$$

holds. Therefore we will still call *Stigler Flexibility* the quantity

$$\mathcal{SF} := \sum_{i=1}^{\infty} \left( \frac{\left(P^i\right)^2}{4a_i} \right) \tag{6.9}$$

It is now clear how to modify the reasonings of Sect.4 in order to apply them to the case of Monopolistic markets.

# 7. Conclusions

We claim to have obtained in this paper the following results:

1) we have characterized the sufficient conditions that have to be satisfied in order to assure the existence of an unique optimal organization structure in dynamic context, both relatively to prices and cost;

2) we have also shown that the increasing price instability allows for a larger region of organization modes which get positive profits;

3) we have underlined the possibility, from an economical point of view, of situations in which non-unique optimal equiprofitable organization modes are present on the same market.

One of the implications of 2) is that if we assume that one price function was an unexpected one .

by a single firm and therefore the chosen organization mode was not the optimal one, then because of price instability, it is still possible (and more likely compared with the case of stable prices) for that firm to have positive profits also if the time-varying price function will be different from the expected one. This implies that we could have different firm organizations that in one industry make positive profits, even if they are not the optimal one. Moreover if no firms could correctly forecast the price oscillation function, it is also possible that no optimal organization actually is present on the market, while there are many which can get positive profits. All these considerations are compatible with the empirical evidence that shows in some industries the coexistence of different organization modes, all getting positive profits, and the absence of a clear evolution towards one optimal organization mode. In a dynamic context and in a situation where the forecast price function could be different from the true one our model allows the theoretical description of quoted possibility of coexistence.

It is finally interesting to compare the statements 1) and 3): in our model we show that only in a deterministic world where firms know exactly the price oscillations and where the convexity properties of the sets of feasible and profitable technologies are suitable the industry will be dominated by one unique organization mode. However if we relax the assumptions of convexity of quoted sets, it will be possible also in a world where firms have perfect knowledge of the price function to have more than one organization structure. Indeed we have shown that, in many industrial sectors, the actual technology is such that is acceptable the hypothesis of non convexity of the set of feasible organization modes.

In the future the work could be developed both from an empirical point of view in order to check which are the industrial sectors where the convexity hypothesis is valid and from the theoretical point of view extending the analysis to duopoly and introducing strategic interactions.

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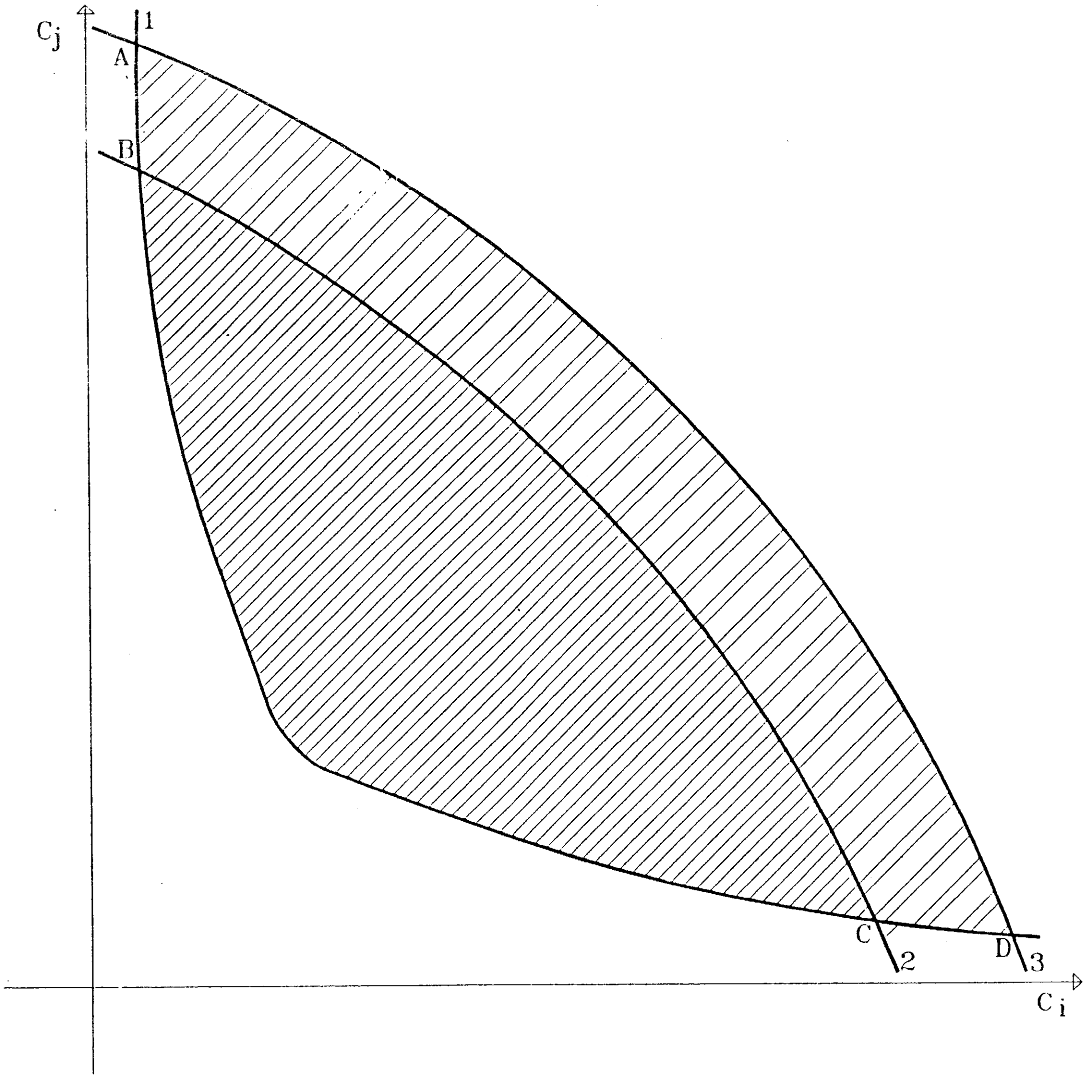


Fig. 1

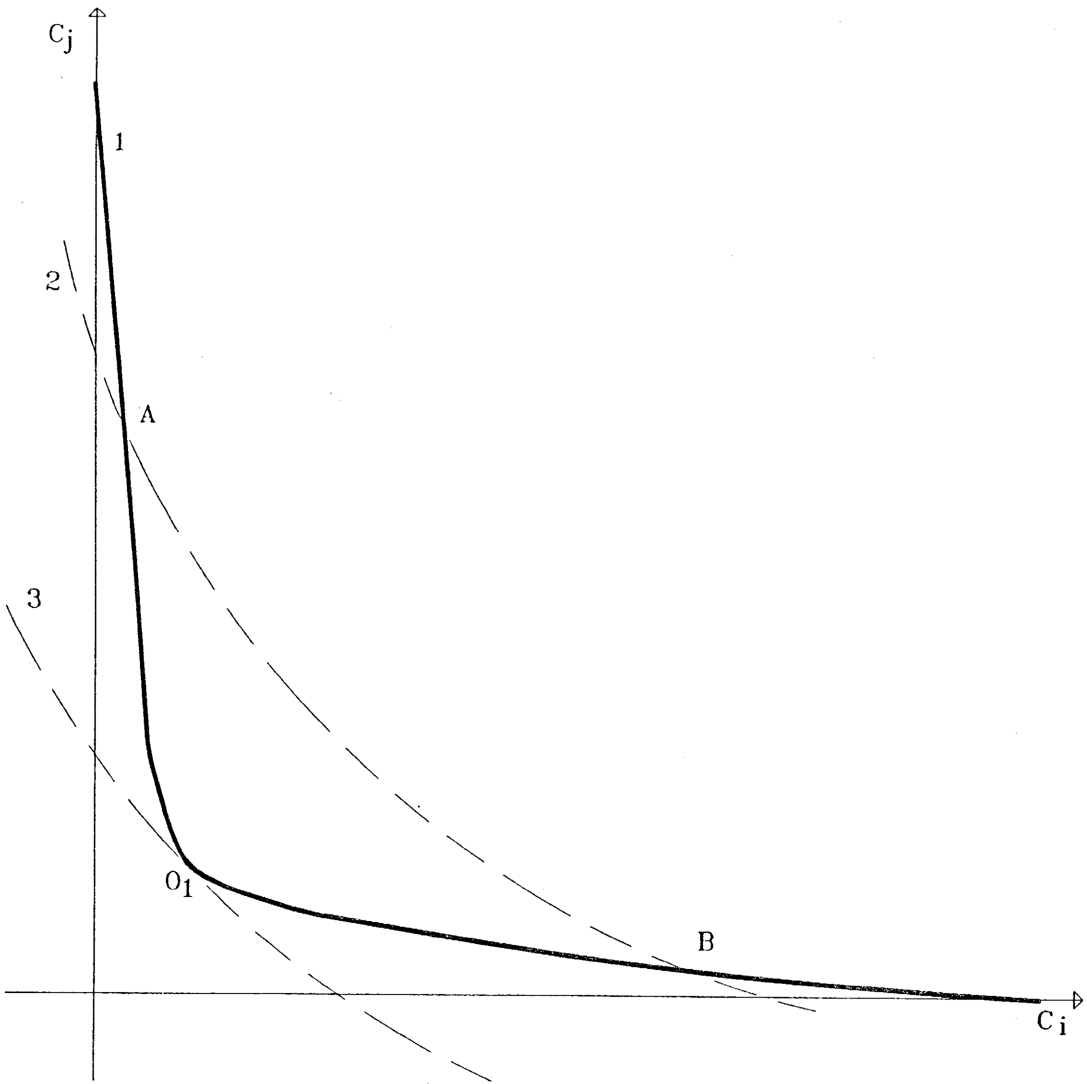


Fig. 2

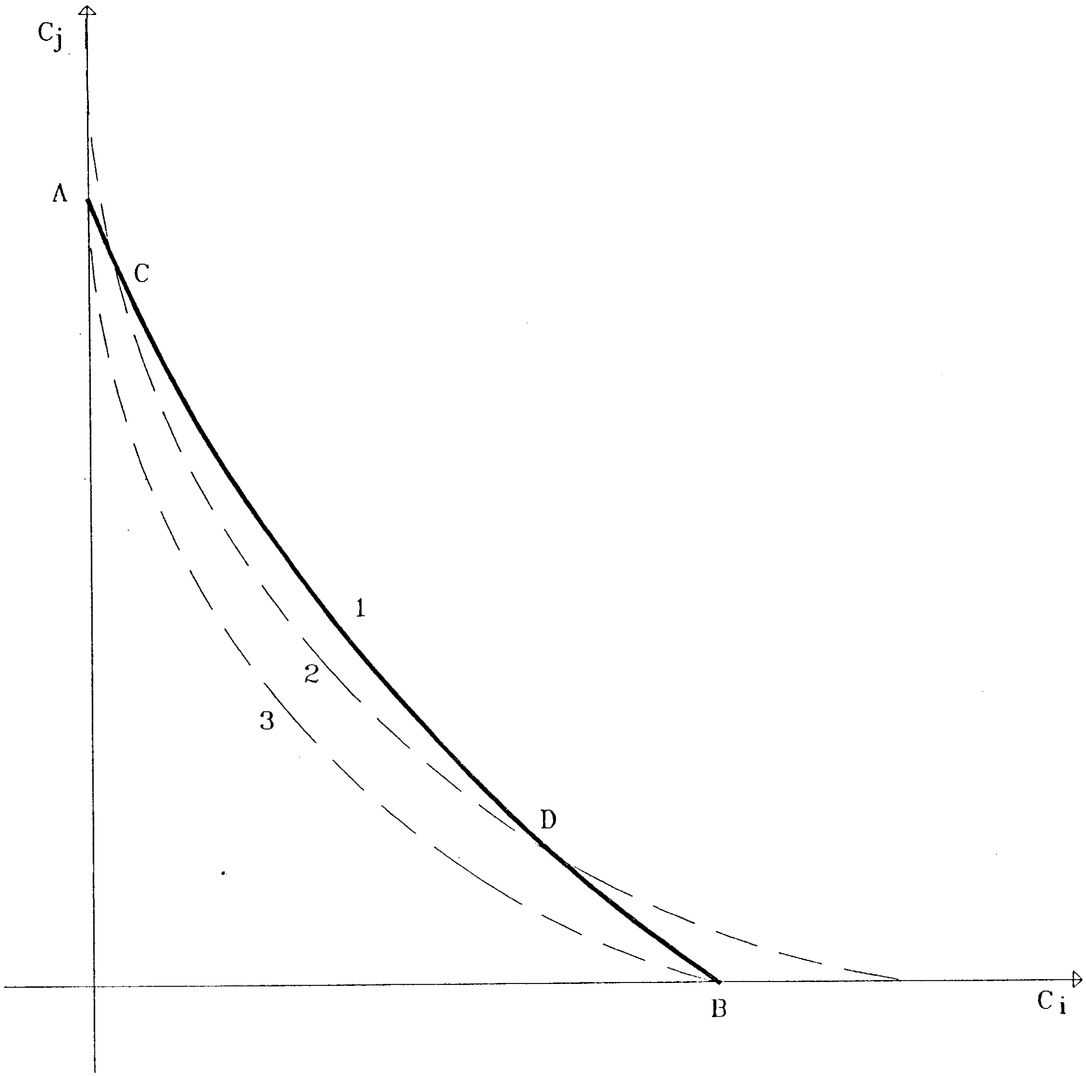


Fig.3

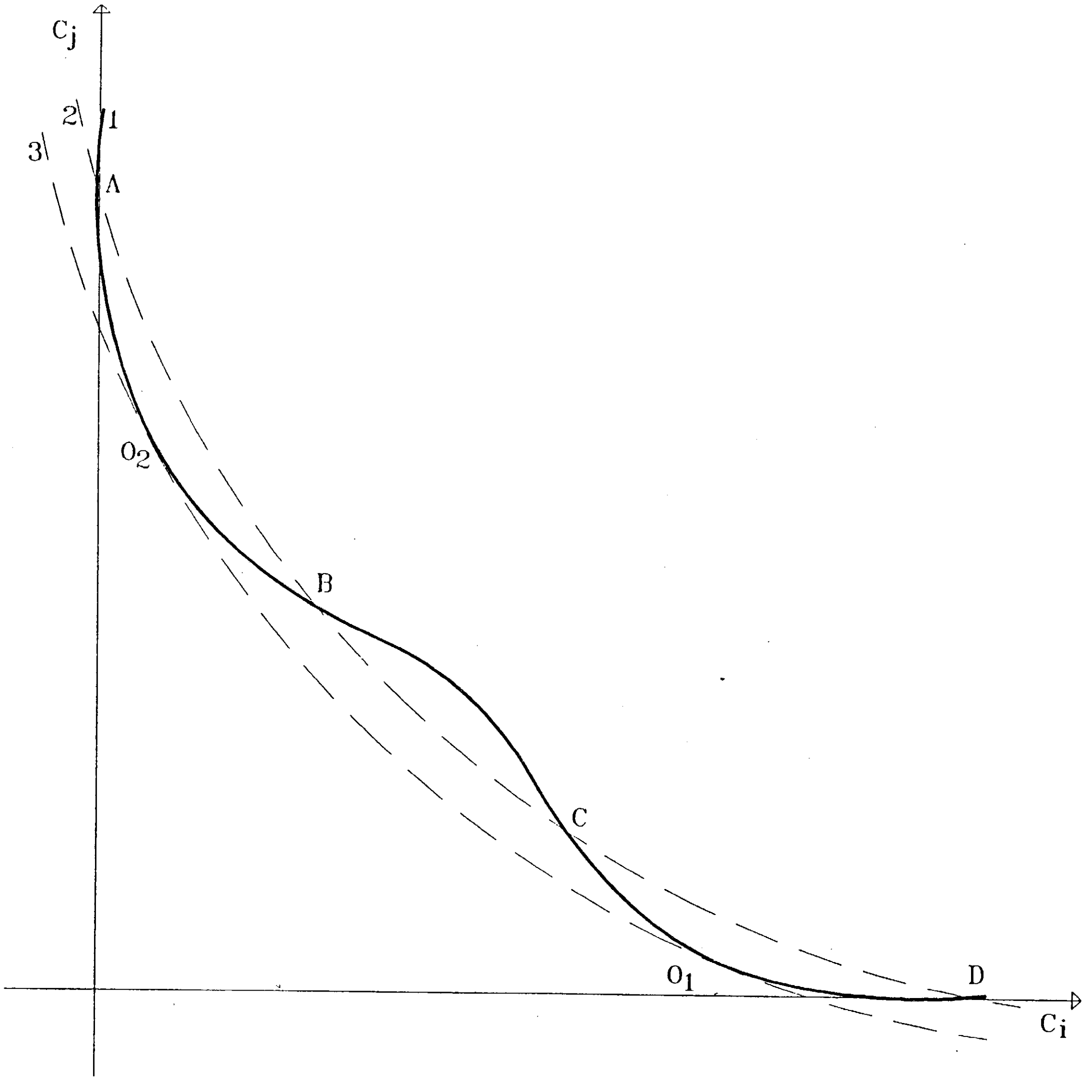


Fig. 4

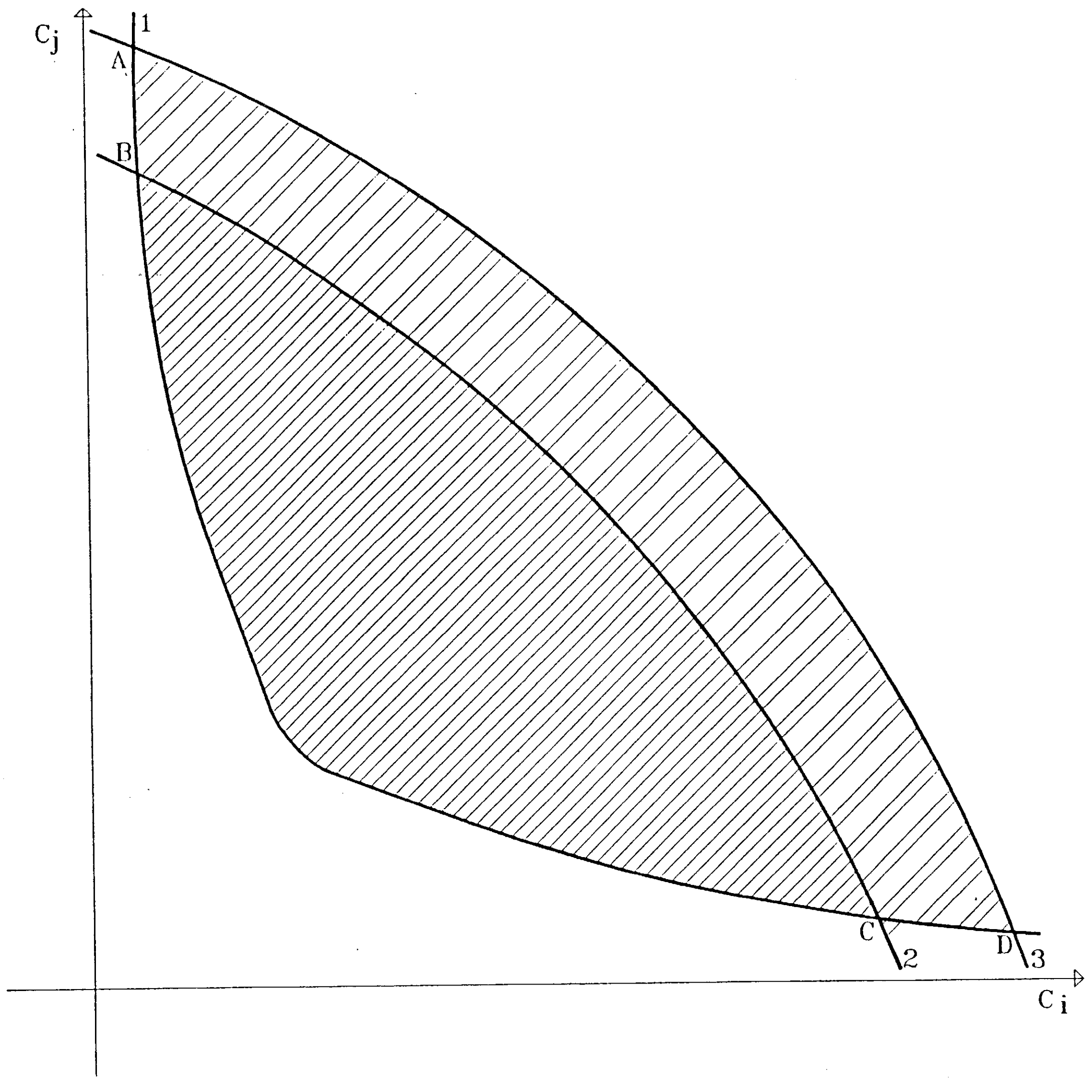


Fig. 1

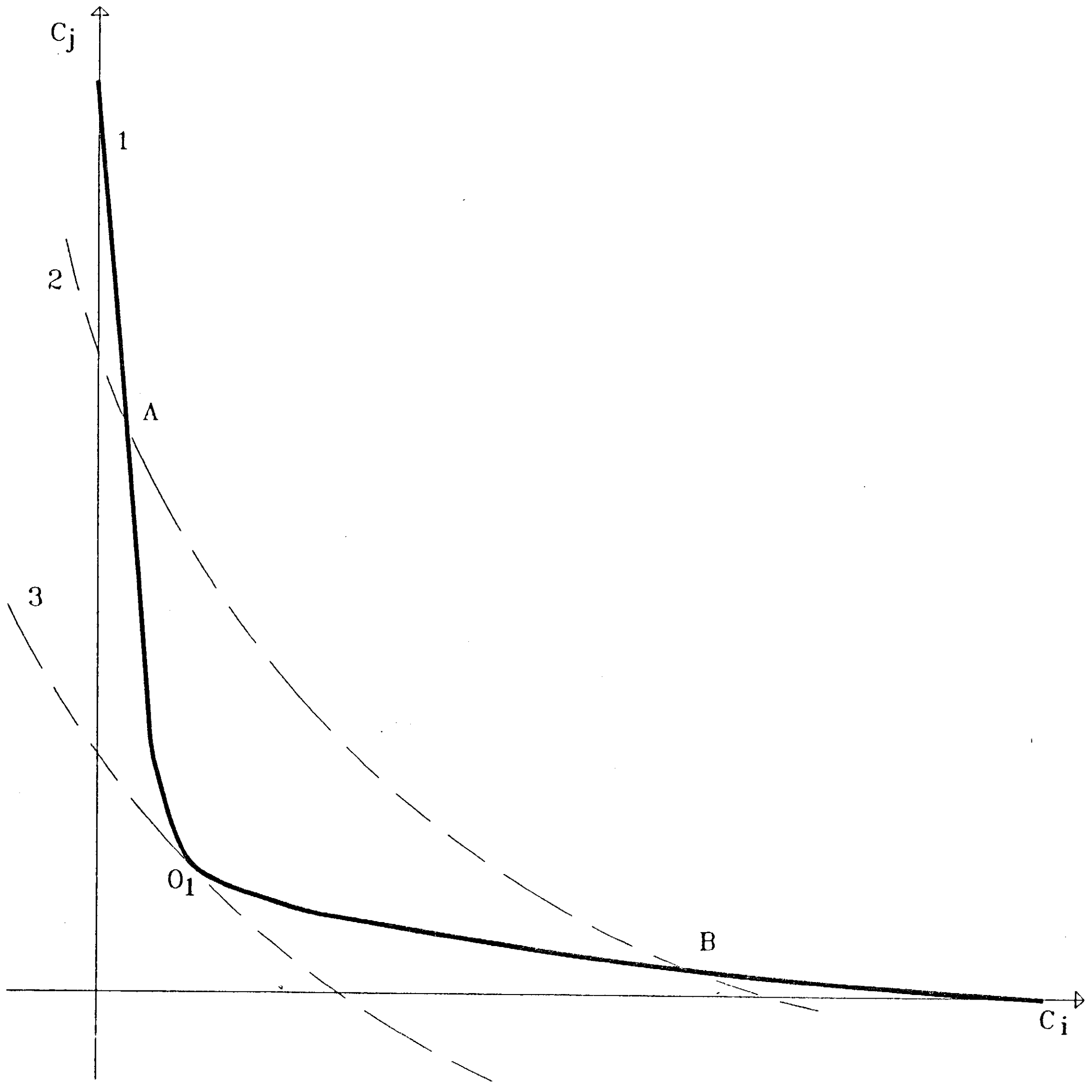


Fig. 2

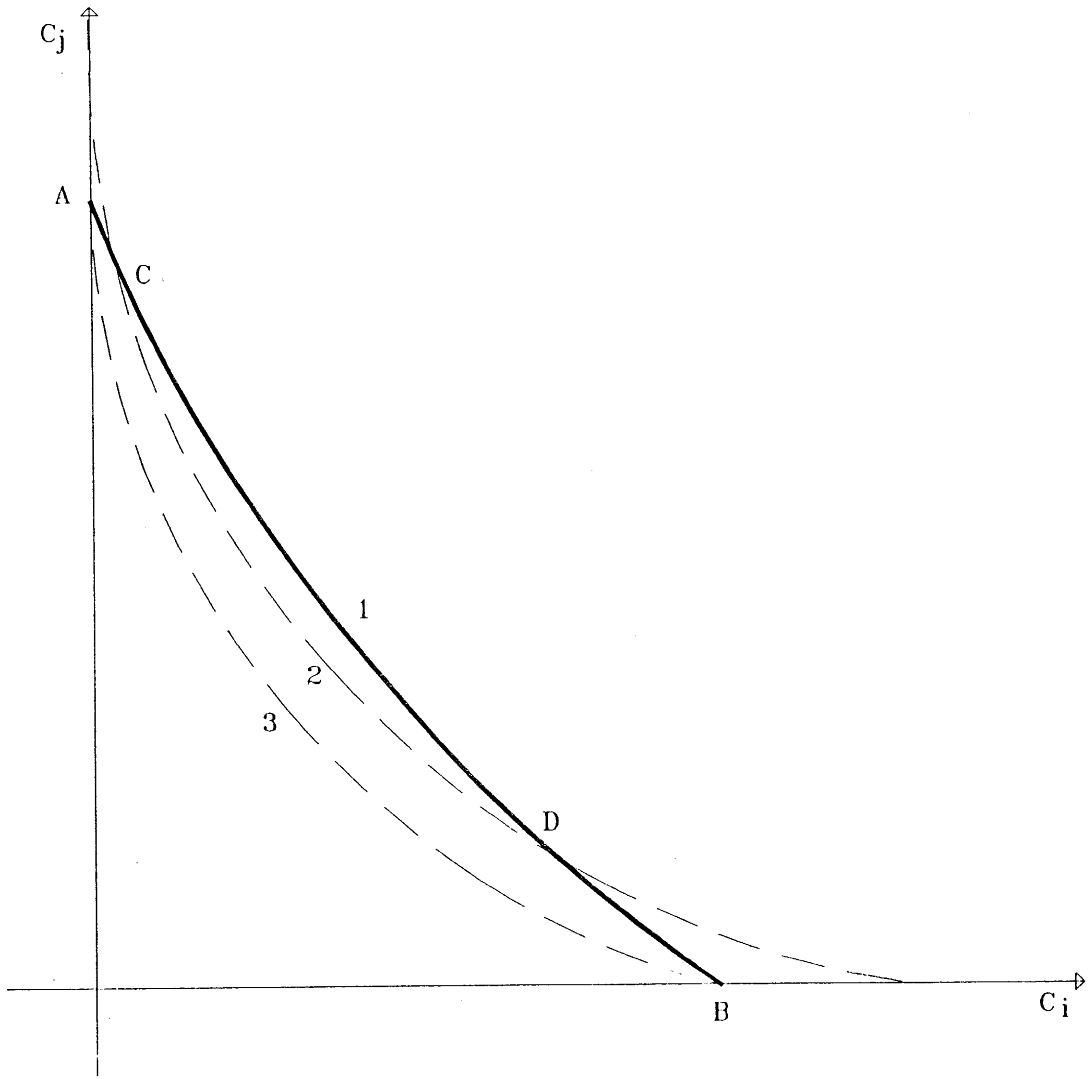


Fig. 3

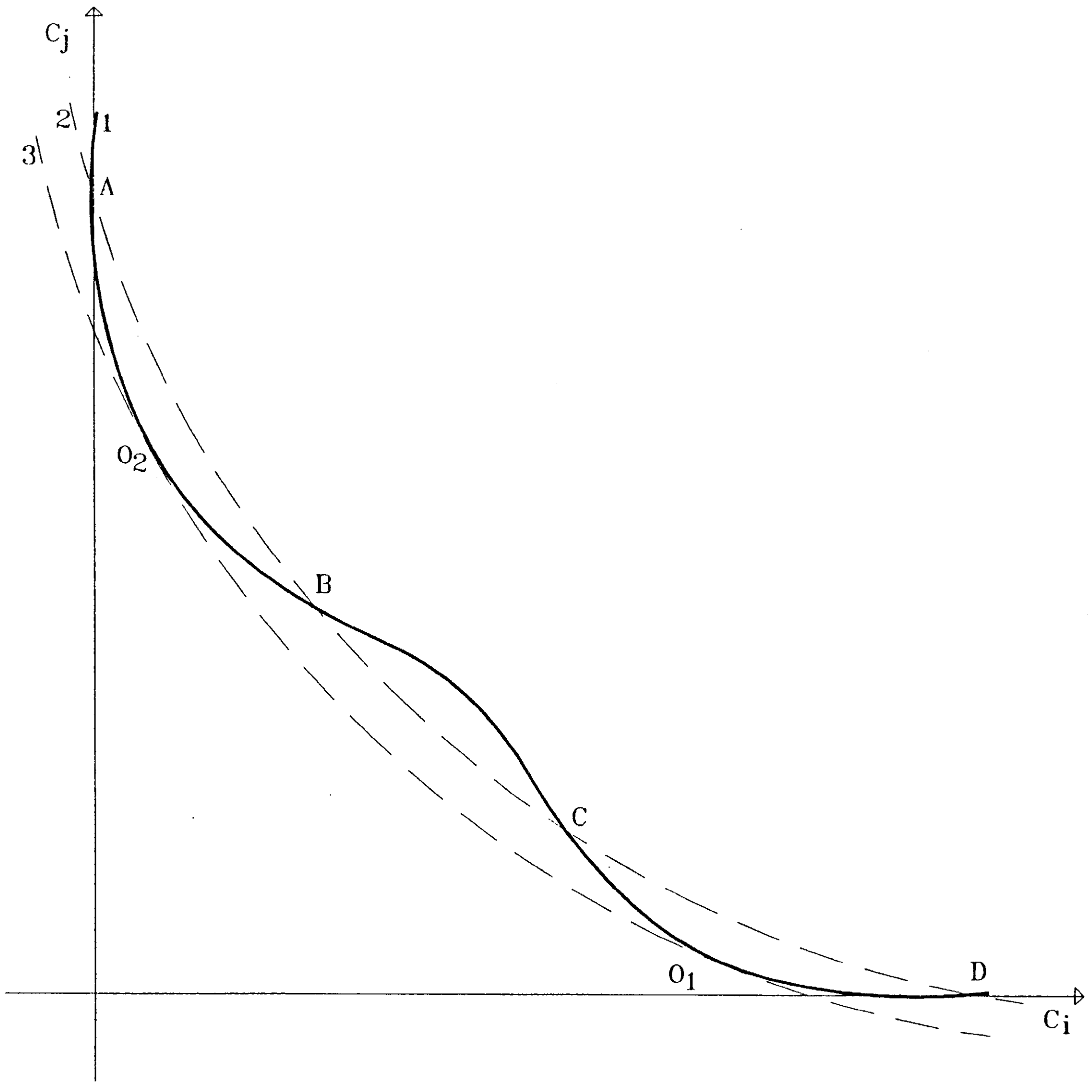


Fig. 4

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